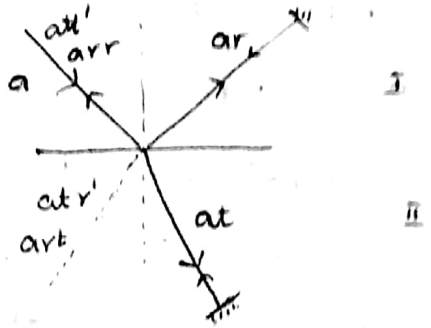
Interference of light

Division of amplitude

- Incident wave is divided to two parts due to partial reflection and refraction.

Stoke's treatment of reflection and refraction

r: reflection co-efficient from I to II
t: transmission co-efficient from I to II
r' & t': reflection and transmission coefficients from II to I.



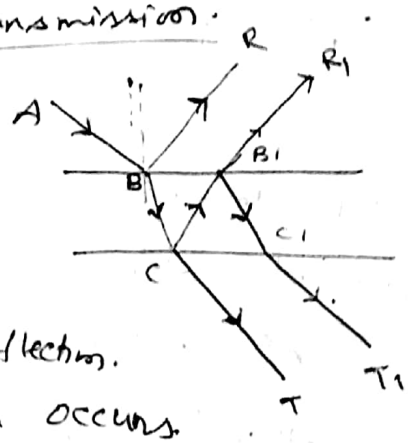
Principle of reversibility:

$arr + att' = a$

$atr' + art = 0 \Rightarrow r = -r'$ and $tt' = 1 - r^2$

Difference of phase by π.

Interference due to reflection and transmission.



BR & B_1R_1 are coherant.
Interference occurs.

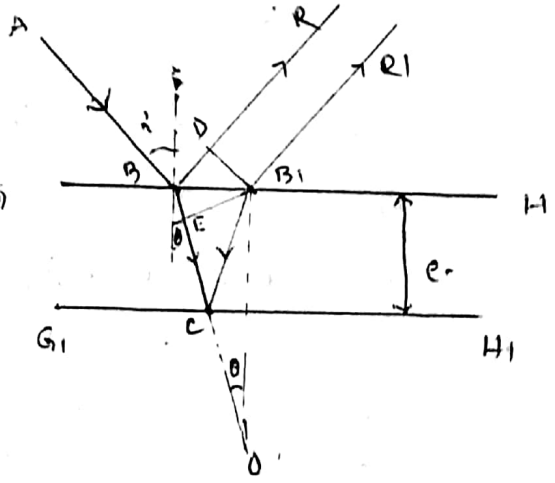
If $\delta = 2n\pi$ (~~maxima~~ minima)
or $\delta = (2n+1)\pi$ (maxima)

a phase change by π occurs due to reflection.

For transmitted light also interference occurs.

Calculation of path difference and phase difference.

Calculation of phase difference



Path diff = Δ

$= (BC + CB_1)_{film} - (BD)_{air}$

$= \mu (BC + CB_1)_{air} - (BD)_{air}$

$= \mu (BE + EC + CB_1) - BD$

$CB_1 = CO$, $EC + CB_1 = EO$

$\mu = \frac{\sin i}{\sin\theta} = \frac{BD/BB_1}{BE/BB_1} = \frac{BD}{BE}$

$BD = \mu BE$

$\therefore \Delta = \mu \times EO = \mu \times 2e\cos\theta$

Phase difference due to path diff = $\delta = \left(\frac{2\pi}{\lambda}\right) \times 2\mu e\cos\theta$.

Phase difference due to reflection = π

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Total Phase difference $\left(\frac{2\pi}{\lambda}\right) 2\mu e\cos\theta + \pi$.

Maxima

$$\frac{2\pi}{\lambda} \times 2\mu e\cos\theta + \pi = 2n\pi$$

$$\Rightarrow \quad 2\mu e\cos\theta = (2n+1)\frac{\lambda}{2}$$

Minima $\quad 2\mu e\cos\theta = n\lambda$

Infinitesimal thin film $\quad e \simeq 0$.

Amplitudes of $B_1R_1 = ar$, $B_1R_1 = at\,r'\,t'$, $B_2R_2 = at\,r'^3\,t'$ ....

$$R = att'r' + att'r'^3 + att'r'^5 + \cdots$$

$$= att'r'\left[1 + r'^2 + r'^4 + \cdots + \infty\right] \qquad r' < 1$$

$$= \frac{att'r'}{(1-r'^2)} = -ar.$$

$R_{min} = ar - ar = 0$

Similarly amplitudes of $CT = att'$, $C_1T_1 = att'r'^2$, $C_2T_2 = att'r'^4$ ....

$$T = att' + att'r'^2 + att'r'^4 + \cdots \infty$$

$$= att'\left[1 + r'^2 + r'^4 + \cdots \infty\right]$$

$$T_{max} = \frac{att'}{(1-r'^2)} = a.$$

$$\therefore \quad T_{max}^2 + T_{min} = a^2.$$

Plane parallel film of finite thickness

$$\Delta = 2\mu e\cos\theta \qquad \& \quad \delta = \frac{2\pi}{\lambda}\Delta.$$

1. Let $\Delta = n\lambda$, then B_1R_1, B_2R_2 ..... all are in same phase
Hence total amplitude of B_1R_1, B_2R_2 ... $= -ar$
So net amplitude $= 0$. So the surface is dark.

2. If $\Delta = (2n+1)\lambda/2$, then B_1R_1 & B_2R_2 are in opposite phase
So the amplitudes of B_1R_1, B_2R_2 .....

$$= -att'r' + att'r'^3 - att'r'^5 + \cdots$$

$$= -att'r'\left(1 - r'^2 + r'^4 - r'^6 + \cdots\right)$$

$$= -\frac{att'r'}{1+r'^2} = \frac{ar(1-r^2)}{1+r^2}$$

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$T_{max} = \frac{2r}{1+r^2}$

Similarly T_{max} = ... and $T_{min} = \frac{2r}{1+r^2}$ where $r = \frac{(n-1)}{(n+1)}$

$\therefore \quad T_{max}^2 + R_{min}^2 = a^2$

and $T_{min}^2 + R_{max}^2 = \left(\frac{2ar}{1+r^2}\right)^2 + \frac{a^2(1-r^2)^2}{(1+r^2)^2}$

$= \frac{4a^2r^2 + a^2(1-r^2)^2}{(1+r^2)^2}$

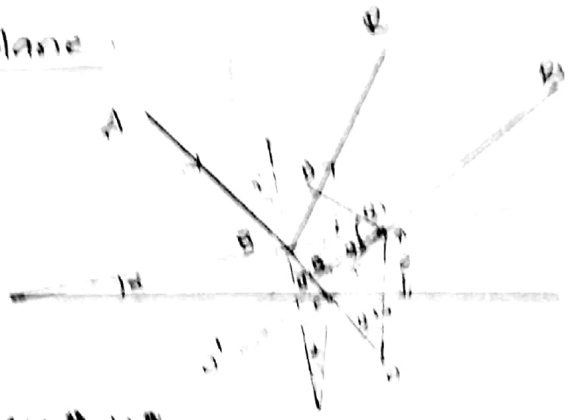
$= \frac{a^2 + 2a^2r^2 + a^2r^4}{(1+r^2)^2} = \frac{a^2(1+r^2)^2}{(1+r^2)^2}$

$= a^2$

Colours of thin film:

In addition to θ and θ, maxima & minima depends on λ also. So maxima for different colours occur in different directions and positions.

Interference due to reflected plane:



Path diff $= \Delta = (BC + CB_1)_{film} - BD_{air}$

$= \mu(BC + CB_1) - BD$

$BD = \mu BE$

$\Delta = \mu(BC + CB_1 - BE)$

$= \mu EO = 2\mu e\cos(\theta + r)$

Localised fringes:

$2\mu e\cos(\theta + r) = (2n+1)\lambda/2$: Maximum brightness

$2\mu e\cos(\theta + r) = n\lambda$: minimum brightness (darkness)

Fringes are formed at O'.

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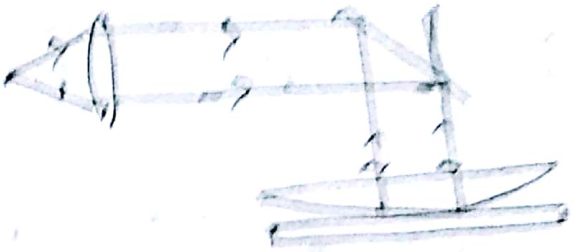
Newton's rings



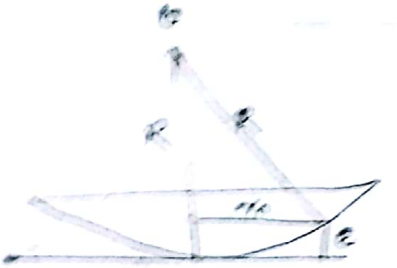
Change of phase by π occurs at C.
If is very small, hence neglected.

$\theta \approx 90°, \quad \cos 90 \simeq 1$

$2\mu e = n\lambda$ (dark fringe)

$2\mu e = (2n+1)\lambda/2$ (bright)

Circular rings.



$R^2 = r_n^2 + (R-e)^2$

$r_n^2 = 2Re - e^2 \simeq 2Re$

$e \simeq \frac{r_n^2}{2R}$

if $r_n^2 = \frac{n\lambda R}{1}$ (dark rings)

$r_n^2 = \frac{(2n+1)\lambda R}{2\mu}$ (bright

$D_n^2 = \frac{4n\lambda R}{\mu}$ (dark), $D_n^2 = \frac{2(2n+1)\lambda R}{\mu}$ (bright)

$D_5 - D_4 = 2\left[\sqrt{5} - \sqrt{4}\right]\sqrt{\lambda R/\mu} = 0.46\sqrt{\lambda R/\mu}$

$D_{17} - D_{16} = 2\left[\sqrt{17} - \sqrt{16}\right]\sqrt{\lambda R/\mu} = 0.24\sqrt{\lambda R/\mu}$.

Fringes are closer as n increases.

Refractive Index of a liquid

wavelength of monochromatic light.

$$\mu = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

Refractive Index of a liquid.

$(D_{n+p}^2)_L - (D_n^2)_L = \frac{4p\lambda R}{\mu}$, $(D_{n+p}^2)_{air} - (D_n^2)_{air} = 4p\lambda R$

$$\mu = \frac{(D_{n+p}^2)_{air} - (D_n^2)_{air}}{(D_{n+p}^2)_{Liquid} - (D_n^2)_{Liq}}$$

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